

Finite Dimensional Linear Systems By Roger W Brockett

It is difficult for me to forget the mild sense of betrayal I felt some ten years ago when I discovered, with considerable dismay, that my two favorite books on linear system theory - Desoer's Notes for a Second Course on Linear Systems and Brockett's Finite Dimensional Linear Systems - were both out of print. Since that time, of course, linear system theory has undergone a transformation of the sort which always attends the maturation of a theory whose range of applicability is expanding in a fashion governed by technological developments and by the rate at which such advances become a part of engineering practice. The growth of the field has inspired the publication of some excellent books; the encyclopedic treatises by Kailath and Chen, in particular, come immediately to mind. Nonetheless, I was inspired to write this book primarily by my practical needs as a teacher and researcher in the field. For the past five years, I have taught a one semester first year graduate level linear system theory course in the School of Electrical Engineering at Cornell. The members of the class have always come from a variety of departments and backgrounds, and consequently have entered the class with levels of preparation ranging from first year calculus and a taste of transform theory on the one extreme to senior level real analysis and abstract algebra on the other.

This dissertation attempts to extend to infinite dimensional linear systems in a Hilbert space some of the stability and stabilizability results that have been obtained for finite dimensional systems using the direct method of Liapunov. The familiar finite dimensional applications of Liapunov's direct method to the stability and stabilizability of linear systems involve the existence of certain positive matrices which satisfy some form of algebraic Riccati equation. Former extensions of these results to infinite dimensional systems in Hilbert space apply exclusively to exponential (uniform asymptotic) stability. Recently, recognizing that exponential stability is a very strong property to expect of some physical systems, some attention has been paid to weaker forms of stability. This thesis generalizes the results of Liapunov's direct method to infinite dimensional systems in a manner that addresses these weaker forms of stability. This is accomplished by developing the distinct concepts of nonnegative, positive, and strictly positive operators. The resulting stability and stabilizability theorems are stated in terms of weak stability, but they are shown to be applicable, in many cases, to strong and exponential stability as well.

The last thirty years have witnessed an enormous effort in the field of robust control of dynamical systems. The main objective of this book is that of presenting, in a unified framework, the main results appeared in the literature on this topic, with particular reference to the robust stability problem for linear systems subject to time-varying uncertainties. The book mainly focuses on those problems for which a definitive solution has been found; indeed most of the results we shall present are given in the form of necessary and sufficient conditions involving the feasibility of Linear Matrix Inequalities based problems. For self-containedness purposes, most of the results provided in the book are proven. We have tried to maintain the development of the proofs as simple as possible, without sacrificing the mathematical rigor. Some parts of the book (especially those contained in Chaps. 2, 3 and 5) can be taught in advanced control courses; however this work is mainly devoted to both researchers in the field of systems and control theory and engineers working in industries which want to apply the methodologies presented in the book to practical control problems. To this regard, as the various results are derived, they are immediately reinforced with real world examples. This book is about the theory of system representations. The systems that are considered are linear, time-invariant, deterministic and finite dimensional. The observation that some representations are more suitable for handling a particular problem than others motivates the

study of representations. In modeling a system, a representation often arises naturally from certain laws that underlie the system. In its most general form the representation then consists of dynamical equations for the system components and of constraint equations reflecting the connection between these components. Depending on the particular problem that is to be investigated, it will sometimes be useful to rewrite the equations, that is, to transform the representation. For this reason it is of special importance to derive transformations that enable one to switch from one representation to another. A new approach of the past decade has been the so-called "behavioral approach" introduced by Willems. One of the main features of the behavioral approach is that it is well suited for modeling the interconnection of systems. It is for this reason that the behavioral approach is a natural choice in the context of modeling. In this book we adopt the behavioral approach: we define a system as a "behavior", that is, a set of trajectories whose mathematical representation by means of differential or difference equations is nonunique. An aspect of this approach that is important in the context of representation theory is the fact that a natural type of equivalence arises.

Special emphasis was given to the following areas: (1) Finite-dimensional linear system theory by algebraic methods: linear systems over a ring, algebraic methods in the study of feedback, polynomial methods for the study of linear systems, $F \text{ mod } G$ invariant subspaces; (2) Infinite-dimensional linear systems: realization theory of infinite-dimensional linear systems; (3) Nonlinear system theory: basic properties of polynomial (discrete-time) systems and their realization theory; and (4) Identification problems in economics: study of econometric model building.

Infinite dimensional systems is now an established area of research. Given the recent trend in systems theory and in applications towards a synthesis of time- and frequency-domain methods, there is a need for an introductory text which treats both state-space and frequency-domain aspects in an integrated fashion. The authors' primary aim is to write an introductory textbook for a course on infinite dimensional linear systems. An important consideration by the authors is that their book should be accessible to graduate engineers and mathematicians with a minimal background in functional analysis. Consequently, all the mathematical background is summarized in an extensive appendix. For the majority of students, this would be their only acquaintance with infinite dimensional systems.

A sequence of 2,400 propositions and problems features only hints. Suitable for advanced undergraduates and graduate students, this unique approach encourages students to work out their own proofs. 1974 edition.

"This book is designed as a graduate text on the mathematical theory of deterministic control. It covers a remarkable number of topics... The book includes material on the realization of both linear and nonlinear systems, impulsive control, and positive linear systemsa "subjects not usually covered in an 'introductory' book... To get so much material in such a short space, the pace of the presentation is brisk. However, the exposition is excellent, and the book is a joy to read. A novel one-semester course covering both linear and nonlinear systems could be given... The book is an excellent one for introducing a mathematician to control theory. The book presents a large amount of material very well, and its use is highly recommended." a "Bulletin of the AMS Mathematical Control Theory: An Introduction presents, in a mathematically precise manner, a unified introduction to deterministic control theory. With the exception of a few more advanced concepts required for the final part of the book, this presentation requires only a knowledge of basic facts from linear algebra, differential equations, and calculus. In addition to classical concepts and ideas, the author covers the stabilization of nonlinear systems using topological methods, realization theory for

nonlinear systems, impulsive control and positive systems, the control of rigid bodies, the stabilization of infinite dimensional systems, and the solution of minimum energy problems. The book will be ideal for a beginning graduate course in mathematical control theory, or for self study by professionals needing a complete picture of the mathematical theory that underlies the applications of control theory.

Many infinite-dimensional linear systems can be modelled in a Hilbert space setting. Others, such as those dealing with heat transfer or population dynamics, need to be set more generally in Banach spaces. This is the first book dealing with well-posed infinite-dimensional linear systems with an input, a state, and an output in a Hilbert or Banach space setting. It is also the first to describe the class of non-well-posed systems induced by system nodes. The author shows how standard finite-dimensional results from systems theory can be extended to these more general classes of systems, and complements them with new results which have no finite-dimensional counterpart. Much of the material presented is original, and many results have never appeared in book form before. A comprehensive bibliography rounds off this work which will be indispensable to all working in systems theory, operator theory, delay equations and partial differential equations.

Based on a streamlined presentation of the authors' successful work *Linear Systems*, this textbook provides an introduction to systems theory with an emphasis on control. Initial chapters present necessary mathematical background material for a fundamental understanding of the dynamical behavior of systems. Each chapter includes helpful chapter descriptions and guidelines for the reader, as well as summaries, notes, references, and exercises at the end. The emphasis throughout is on time-invariant systems, both continuous- and discrete-time.

The theory of linear damped oscillations was originally developed more than hundred years ago and is still of vital research interest to engineers, mathematicians and physicists alike. This theory plays a central role in explaining the stability of mechanical structures in civil engineering, but it also has applications in other fields such as electrical network systems and quantum mechanics. This volume gives an introduction to linear finite dimensional damped systems as they are viewed by an applied mathematician. After a short overview of the physical principles leading to the linear system model, a largely self-contained mathematical theory for this model is presented. This includes the geometry of the underlying indefinite metric space, spectral theory of J -symmetric matrices and the associated quadratic eigenvalue problem. Particular attention is paid to the sensitivity issues which influence numerical computations. Finally, several recent research developments are included, e.g. Lyapunov stability and the perturbation of the time evolution.

This book discusses the realization and control problems of finite-dimensional dynamical systems which contain linear and nonlinear systems. The author focuses on algebraic methods for the discussion of control problems of linear and non-linear dynamical systems. The book contains detailed examples to showcase the effectiveness of the presented method. The target audience comprises primarily research experts in the field of control theory, but the book may also be beneficial for graduate students alike.

This book is the result of our teaching over the years an undergraduate course on Linear Optimal Systems to applied mathematicians and a first-year graduate course on Linear

Systems to engineers. The contents of the book bear the strong influence of the great advances in the field and of its enormous literature. However, we made no attempt to have a complete coverage. Our motivation was to write a book on linear systems that covers finite dimensional linear systems, always keeping in mind the main purpose of engineering and applied science, which is to analyze, design, and improve the performance of physical systems. Hence we discuss the effect of small nonlinearities, and of perturbations of feedback. It is our hope that the book will be a useful reference for a first-year graduate student. We assume that a typical reader with an engineering background will have gone through the conventional undergraduate single-input single-output linear systems course; an elementary course in control is not indispensable but may be useful for motivation. For readers from a mathematical curriculum we require only familiarity with techniques of linear algebra and of ordinary differential equations.

This book presents a unified algebraic approach to stabilization problems of linear boundary control systems with no assumption on finite-dimensional approximations to the original systems, such as the existence of the associated Riesz basis. A new proof of the stabilization result for linear systems of finite dimension is also presented, leading to an explicit design of the feedback scheme. The problem of output stabilization is discussed, and some interesting results are developed when the observability or the controllability conditions are not satisfied. Originally published in 1970, Finite Dimensional Linear Systems is a classic textbook that provides a solid foundation for learning about dynamical systems and encourages students to develop a reliable intuition for problem solving. The theory of linear systems has been the bedrock of control theory for 50 years and has served as the springboard for many significant developments, all the while remaining impervious to change. Since linearity lies at the heart of much of the mathematical analysis used in applications, a firm grounding in its central ideas is essential. This book touches upon many of the standard topics in applied mathematics, develops the theory of linear systems in a systematic way, making as much use as possible of vector ideas, and contains a number of nontrivial examples and many exercises.

This monograph deals with approximation and noise cancellation of dynamical systems which include linear and nonlinear input/output relations. It will be of special interest to researchers, engineers and graduate students who have specialized in filtering theory and system theory. From noisy or noiseless data,

reduction will be made. A new method which reduces noise or model information will be proposed. Using this method will allow model description to be treated as noise reduction or model reduction. As proof of the efficacy, this monograph provides new results and their extensions which can also be applied to nonlinear dynamical systems. To present the effectiveness of our method, many actual examples of noise and model information reduction will also be provided. Using the analysis of state space approach, the model reduction problem may have become a major theme of technology after 1966 for emphasizing efficiency in the fields of control, economy, numerical analysis, and others. Noise reduction problems in the analysis of noisy dynamical systems may

have become a major theme of technology after 1974 for emphasizing efficiency in control. However, the subjects of these researches have been mainly concentrated in linear systems. In common model reduction of linear systems in use today, a singular value decomposition of a Hankel matrix is used to find a reduced order model. However, the existence of the conditions of the reduced order model are derived without evaluation of the resultant model. In the common typical noise reduction of linear systems in use today, the order and parameters of the systems are determined by minimizing information criterion. Approximate and noisy realization problems for input/output relations can be roughly stated as follows: A. The approximate realization problem. For any input/output map, find one mathematical model such that it is similar

to the input/output map and has a lower dimension than the given minimal state space of a dynamical system which has the same behavior to the input/output map. B. The noisy realization problem.

My aim, in writing this monograph, has been to remedy this omission by presenting a comprehensive and unified theory of observers for continuous-time and discrete-time linear systems. The book is intended for post-graduate students and researchers specializing in control systems, now a core subject in a number of disciplines. Forming, as it does, a self-contained volume it should also be of service to control engineers primarily interested in applications, and to mathematicians with some exposure to control problems.

Finite Dimensional Linear Systems SIAM

"There are three words that characterize this work: thoroughness, completeness and clarity. The authors are congratulated for taking the time to write an excellent linear systems textbook!" —IEEE Transactions on Automatic Control
Linear systems theory plays a broad and fundamental role in electrical, mechanical, chemical and aerospace engineering, communications, and signal processing. A thorough introduction to systems theory with emphasis on control is presented in this self-contained textbook, written for a challenging one-semester graduate course. A solutions manual is available to instructors upon adoption of the text. The book's flexible coverage and self-contained presentation also make it an excellent reference guide or self-study manual. For a treatment of linear systems that focuses primarily on the time-invariant case using streamlined presentation of the material with less formal and more intuitive proofs, please see the authors' companion book entitled *A Linear Systems Primer*.

One of the main problems in control theory is the stabilization problem consisting of finding a feedback control law ensuring stability; when the linear approximation is considered, the natural problem is stabilization of a linear system by linear state feedback or by using a linear dynamic controller. This problem was intensively studied during the last decades and many important results have been obtained. The present monograph is based mainly on results obtained by the authors. It focuses on stabilization of systems with slow and fast motions, on stabilization procedures that use only poor information about the system (high-gain stabilization and adaptive stabilization), and also on discrete time implementation of the stabilizing procedures. These topics are important in many applications of stabilization theory. We hope that this monograph may illustrate the way in which mathematical theories do influence advanced technology. This book is not intended to be a text book nor a guide for control-designers. In engineering practice, control-design is a very complex task in which stability is only one of the requirements and many aspects and facets of the problem have to be taken into consideration. Even if we restrict ourselves to stabilization, the book does not provide just recipes, but it focuses more on the ideas lying behind the recipes. In short, this is not a book on control, but on some mathematics of control.

This report deals with research results obtained in the following areas: (1) Finite-dimensional linear system theory by algebraic methods--linear systems over a ring, algebraic methods in the study of feedback, polynomial methods for the study of linear systems, $F \text{ mod } G$ invariant subspaces; (2) Infinite-dimensional linear systems--realization theory of infinite-dimensional linear systems; (3) Nonlinear system theory--basic properties of polynomial (discrete-time) systems and their realization theory; and (4) Identification problems in economics--study of econometric model building.

A treatment of system theory within the context of finite dimensional spaces, this text is appropriate for students with no previous experience of operator theory. The three-part approach, with notes and references for each section, covers linear algebra and finite dimensional systems, operators in Hilbert space, and linear systems in Hilbert space. 1981 edition.

This monograph deals with control problems of discrete-time dynamical systems which include linear and nonlinear input/output relations. In its present second enlarged edition the control problems of linear and non-linear dynamical systems will be solved as algebraically as possible. Adaptive control problems are newly proposed and solved for dynamical systems which satisfy the time-invariant condition. The monograph provides new results and their extensions which can also be more applicable for nonlinear dynamical systems. A new method which produces manipulated inputs is presented in the sense of state control and output control. To present the effectiveness of the method, many numerical examples of control problems are provided as well.

Three-part approach, with notes and references for each section, covers linear algebra and finite dimensional systems, operators in Hilbert space, and linear systems in Hilbert space. 1981 edition.

Geared primarily to an audience consisting of mathematically advanced undergraduate or beginning graduate students, this text may additionally be used by engineering students interested in a rigorous, proof-oriented systems course that goes beyond the classical frequency-domain material and more applied courses. The minimal mathematical background required is a working knowledge of linear algebra and differential equations. The book covers what constitutes the common core of control theory and is unique in its emphasis on foundational aspects. While covering a wide range of topics written in a standard theorem/proof style, it also develops the necessary techniques from scratch. In this second edition, new chapters and sections have been added, dealing with time optimal control of linear systems, variational and numerical approaches to nonlinear control, nonlinear controllability via Lie-algebraic methods, and controllability of recurrent nets and of linear systems with bounded controls.

Linear algebra forms the basis for much of modern mathematics—theoretical, applied, and computational. Finite-Dimensional Linear Algebra provides a solid foundation for the study of advanced mathematics and discusses applications of linear algebra to such diverse areas as combinatorics, differential equations, optimization, and approximation. The author begins with an overview of the essential themes of the book: linear equations, best approximation, and diagonalization. He then takes students through an axiomatic development of vector spaces, linear operators, eigenvalues, norms, and inner products. In addition to discussing the special properties of symmetric matrices, he covers the Jordan canonical form, an important theoretical tool, and the singular value decomposition, a powerful tool for computation. The final chapters present introductions to numerical linear algebra and analysis in vector spaces, including a brief introduction to functional analysis (infinite-dimensional linear algebra). Drawing on material from the author's own course, this textbook gives students a strong theoretical understanding of linear algebra. It offers many illustrations of how linear algebra is used throughout mathematics.

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